

**GANDHI INSTITUTE OF ENGINEERING AND TECHNOLOGY, ODISHA, GUNUPUR  
(GIET UNIVERSITY)**



B. Tech (Third Semester - Regular) Examinations, November – 2024

**23BBSBS23001– Engineering Mathematics-III**

(Electronics and Communication Engineering)

Time: 3 hrs

Maximum: 60 Marks

**Answer ALL questions**

(The figures in the right hand margin indicate marks)

**PART – A**

**(2 x 5 = 10 Marks)**

Q.1. Answer **ALL** questions

- |                                                                                                                                                                                             | CO # | Blooms<br>Level |
|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|------|-----------------|
| a. Determine whether the function $2xy + i(x^2 - y^2)$ is analytic or not.                                                                                                                  | CO1  | K1              |
| b. Find the zeros of $f(z) = \frac{z^2 + 1}{1 - z^2}$ .                                                                                                                                     | CO2  | K1              |
| c. Find the divided difference of $f(x) = x^3 + x + 2$ for the arguments 1,3,6 and 11.                                                                                                      | CO3  | K2              |
| d. In a particular manufacturing process, it is found that, on the average 1% of the items is defective. What is the probability that the fifth item inspected is the first defective item? | CO4  | K2              |
| e. Consider a Markov chain with state $\{0,1\}$ and transition probability matrix $P = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ . Is the state 0 periodic? If so what is the period?   | CO5  | K2              |

**PART – B**

**(10 x 5 = 50 Marks)**

Answer **ALL** the questions

- |                                                                                                                                                                                                   | Marks | CO # | Blooms<br>Level |
|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-------|------|-----------------|
| 2. a. If $f(z) = e^z$ then show that $u$ and $v$ are harmonic functions. [ $z = x + iy$ and $f(z) = u + iv$ ].                                                                                    | 5     | CO1  | K3              |
| b. Determine the region D of the $w$ -plane into which the triangular region D enclosed by the lines $x = 0$ , $y = 0$ and $x + y = 1$ is transformed under the transformation $w = 2z$ .<br>(OR) | 5     | CO1  | K4              |
| c. Find the bilinear transformation that maps the points $z = 0, -1, i$ into the points $w = i, 0, \infty$ respectively.                                                                          | 5     | CO1  | K3              |
| d. Prove that an analytic function with constant modulus is constant.                                                                                                                             | 5     | CO1  | K4              |
| 3.a. Using Cauchy's integral formula, evaluate $\int_C \frac{z+1}{(z-3)(z-1)} dz$ where C is the circle $ z  = 2$ .                                                                               | 5     | CO2  | K3              |
| b. Find the Laurent's series expansion of the function $\frac{z-1}{(z+2)(z+3)}$ , valid in the region $2 <  z  < 3$ .<br>(OR)                                                                     | 5     | CO2  | K4              |
| c. Find the Taylor's series for $\frac{1}{z-2}$ about $z=1$ up to first four terms.                                                                                                               | 5     | CO2  | K3              |
| d. Evaluate using Cauchy's Residue theorem, $\int_C \frac{\sin \pi z^2 + \cos \pi z^2}{(z-1)(z-2)} dz$ where $C :  z  = 3$ .                                                                      | 5     | CO2  | K4              |

- 4.a. Find the real root of  $x^3 - 2x - 5 = 0$  using Newton Raphson's method.[up to three decimal places] 5 CO3 K3
- b. Using Lagrange's interpolation, find  $f(3)$  from the following data. 5 CO3 K3
- |        |   |   |    |     |
|--------|---|---|----|-----|
| $x$    | 0 | 1 | 2  | 5   |
| $f(x)$ | 2 | 3 | 12 | 147 |
- (OR)
- c. Find the first derivative of  $(x)^{\frac{1}{3}}$  at  $x = 50$  and  $x = 56$  given the table below: 5 CO3 K3
- |                         |        |        |        |        |        |        |        |
|-------------------------|--------|--------|--------|--------|--------|--------|--------|
| $x$                     | 50     | 51     | 52     | 53     | 54     | 55     | 56     |
| $y = (x)^{\frac{1}{3}}$ | 3.6840 | 3.7084 | 3.7325 | 3.7563 | 3.7798 | 3.8030 | 3.8259 |
- d. Solve for a positive root of  $x^3 - 4x + 1 = 0$  by Regula Falsi method.[up to three decimal places] 5 CO3 K3
- 5.a. The incidence of occupational disease in an industry is such that the workers have a 20% chance of suffering from it. What is the probability that out of six workers 4 or more will contract disease? 5 CO4 K4
- b. If  $X$  is uniformly distributed over  $(0,10)$ , find i)  $P(X < 4)$  ii)  $P(X > 6)$  5 CO4 K4
- iii)  $P(2 < X < 5)$
- (OR)
- c. By the method of least squares find the best fitting straight line to the data given below: 5 CO4 K3
- |     |    |    |    |    |    |
|-----|----|----|----|----|----|
| $x$ | 5  | 10 | 15 | 20 | 25 |
| $y$ | 15 | 19 | 23 | 26 | 30 |
- d. A company keeps records of accidents. During a recent safety review, a random sample of 60 accidents was selected and classified by the day of the week on which they occurred. 5 CO4 K4
- |                     |     |      |     |       |     |
|---------------------|-----|------|-----|-------|-----|
| Day                 | Mon | Tues | Wed | Thurs | Fri |
| Number of accidents | 8   | 12   | 9   | 14    | 17  |
- Test whether there is any evidence that accidents are more likely on some days than others.
- 6.a. Prove that the random process  $X(t) = A \cos(\omega t + \theta)$ , where  $A, \omega$  are constants and  $\theta$  is uniformly distributed over  $(0, 2\pi)$  is a wide sense stationary process. 5 CO5 K4
- b. Suppose that customers arrive at a bank according to a Poisson process with a mean rate of 3 per minute, find the probability that during a time interval of 2 minute (i) more than 4 customers arrive (ii) exactly 4 customers arrive 5 CO5 K4
- (OR)
- c. The transition probability matrix of a Markov chain  $\{X_n\}$ ,  $n = 1, 2, \dots$  having 3 states 1, 2 and 3 is  $P = \begin{bmatrix} 0.1 & 0.5 & 0.4 \\ 0.6 & 0.2 & 0.2 \\ 0.3 & 0.4 & 0.3 \end{bmatrix}$  and the initial distribution is  $P^{(0)} = (0.7 \ 0.2 \ 0.1)$ . Find  $P = (X_3 = 2, X_2 = 3, X_1 = 3, X_0 = 2)$ . 5 CO5 K3
- d. If  $\{X(t)\}$  is a wide-sense stationary process with autocorrelation  $R(\tau) = Ae^{-\alpha|\tau|}$ , determine the second-order moment of the random variable  $X(8) - X(5)$ . 5 CO5 K4

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