GANDHI INSTITUTE OF ENGINEERING AND TECHNOLOGY, ODISHA, GUNUPUR
(GIET UNIVERSITY)

B. Tech (Third Semester - Regular) Examinations, November – 2024

23BCSBS23001 - Discrete Mathematics

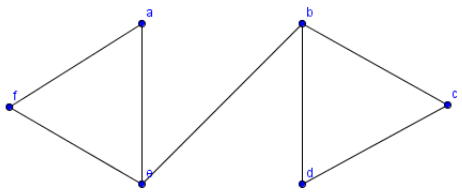
(CSE)

Time: 3 hrs Maximum: 60 Marks

Answer ALL questions
(The figures in the right hand margin indicate marks)

PART – A **(2 x 5 = 10 Marks)**

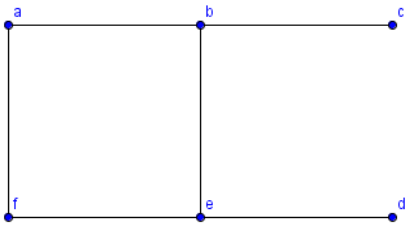
Q.1. Answer *ALL* questions	**CO #**	**Blooms Level**
a. Define Quantifiers with an example.	CO1	K2
b. If $A = (1,3)$ and $B = (2,4,5)$, then how many relations can be formed from A to B ? Write the relations which have two ordered pairs among them.	CO2	K3
c. Define a Boolean function with suitable examples.	CO3	K3
d. Prove that any group of order p^2 where p is a prime, is abelian.	CO4	K1
e. Define coloring. Find the chromatic number for the following graph.	CO5	K2



PART – B **(10 x 5 = 50 Marks)**

Answer *ALL* the questions	**Marks**	**CO #**	**Blooms Level**
2. a. Prove that $(p \wedge q) \Leftrightarrow p$ is logical equivalence to $p \Rightarrow q$.	5	CO1	K2
b. Use mathematical induction to show that $1 + 3^2 + 5^2 + 7^2 + \cdots + (2n-1)^2 = \frac{1}{3} n(2n-1)(2n+1)$.	5	CO1	K2
(OR)			
c. Determine whether the following formula is tautology, contradiction, or neither. $(p \Rightarrow \bar{q}) \wedge (p \wedge q)$	5	CO1	K3
d. Let P(x) be the predicate "x is divisible by 3" where x is an integer. Determine the truth value of $\forall x$ P(x) and $\exists x$ P(x).	5	CO1	K1
3.a. Show that the sequence $\{a_n\}$ is a solution of the recurrence relation $a_n = a_{n-1} + 2a_{n-2} + 2n - 9$ if $a_n = 5(-1)^n - n + 2$.	5	CO2	K4
b. In a survey of 540 college students, it is found that 128 like carrot, 188 like beetroot, 116 like cauliflower, 52 like both carrot and beetroot, 56 like both carrot and cauliflower, 44 like both beetroot and cauliflower and 28 like all three vegetables. How many of the 540 students do not like any of these vegetables.	5	CO2	K3
(OR)			
c. Let R be a binary relation on the set of all positive integer and such that $R = \{(a,b) \mid a = \sqrt{b}\}$. Is R Reflexive? Symmetric? Anti-symmetric? Transitive? an Equivalence relation? A partial ordering relation?	5	CO2	K2

d.	A large software development company employs 100 computer programmers. Of them, 45 are proficient in Java, 30 in C++, 20 in Python, six in C++ and Java, one in Java and Python, five in C++ and Python, and just one programmer is proficient in all three languages above. Determine the number of computer programmers that are not proficient in any of these three languages.	5	CO2	K2
4.a.	Consider the Boolean algebra D_{210} (D-Divisor). i) List its elements and draw its diagram. ii) Find the set A of atoms. iii)Find two sub algebras with eight elements.	5	CO3	K4
b.	Prove that every chain is a distributive lattice.	5	CO3	K4
	(OR)			
c.	In any Boolean algebra, show that $a = b$ if and only if $a\bar{b} + \bar{a}b = 0$.	5	CO3	K3
d.	Prove that every distributive lattice is modular.	5	CO3	K4
5.a.	Consider the group $G = \{1,2,3,4,5,6,7\}$ under multiplication modulo 8. i) Give the Cayley Table of G ii) Find $2^{-1}, 3^{-1}, 6^{-1}$ iii)Find the order of the subgroups generated by 2 and 3. iv)Is $(Z_n, \otimes)$a group for any n? Justify your answer?	5	CO4	K1
b.	Determine whether the set $S = \{1,2,3,6,12,24\}$, with the binary operation $a * b = LCM(a,b)$ is a semigroup, a monoid or neither.	5	CO4	K2
	(OR)			
c.	Show that the set $S = \{1,5,7,11\}$ is a group with respect to multiplication modulo 12.	5	CO4	K1
d.	Let H and K be subgroups of a group G. Prove that $H \cap K$ is subgroup of G.	5	CO4	K3
6.a.	Answer all the questions: Justify the answer. i) Determine whether the following sequence is graphic. 5, 4, 3, 2, 1. ii) Find the degree sequence of K_5. iii)How many vertices does a regular graph of degree four with 10 edges have?	5	CO5	K2
b.	Prove that in a tree, there exist a unique path between pair of vertices.	5	CO5	K3
	(OR)			
c.	Write the adjacent and incident matrices of the following graph.	5	CO5	K2



d.	Derive Euler's formula for planar graph.	5	CO5	K3

--- End of Paper ---